MATH 2B: LINEAR ALGEBRA

Foothill College Course Outline of Record

Heading	Value
Effective Term:	Summer 2024
Units:	5
Hours:	5 lecture per week (60 total per quarter)
Prerequisite:	MATH 1C.
Advisory:	Demonstrated proficiency in English by placement via multiple measures OR through an equivalent placement process OR completion of ESLL 125 & ESLL 249.
Degree & Credit Status:	Degree-Applicable Credit Course
Foothill GE:	Non-GE
Transferable:	CSU/UC
Grade Type:	Letter Grade (Request for Pass/No Pass)
Repeatability:	Not Repeatable

Student Learning Outcomes

- Students will be able to create, interpret, analyze, and discuss mathematical models of physical problems using linear algebraic techniques.
- Students will develop conceptual understanding of the four major problems in introductory linear algebra: the matrix-multiplication problem, the linear systems problem, the least-squares problem, and the eigenvalue/eigenvector problem. Students will demonstrate and communicate this understanding by reasoning with definitions and theorems and connecting concepts.
- Students will solve each of the major problems (the matrix-multiplication problem, the linear systems problem, the least-squares problem, and the eigenvalue/eigenvector problem) using appropriate methods.

Description

A first course in linear algebra, including systems of linear equations, matrices, linear transformations, determinants, abstract vector spaces and subspaces, eigenvalues and eigenvectors, inner product spaces and orthogonality, and selected applications of these topics.

Course Objectives

The student will be able to:

1. Solve linear systems using various methods in linear algebra, and analyze the systems.
2. Demonstrate an understanding of matrix operations, their properties, and various characterizations of invertible matrices including the Invertible Matrix Theorem.
3. Evaluate the determinant and demonstrate an understanding of its properties.
4. Demonstrate an understanding of vector spaces and subspaces, identify those spaces and understand their characterizations.
5. Demonstrate an understanding of eigenvectors, eigenvalues, and their usage in many fields.
6. Demonstrate an understanding of an orthogonal projection of a vector onto a subspace, and solve related problems in linear algebra.
7. Write linear systems to model phenomena from various real-life problems, and discuss their solutions to demonstrate an understanding of applications of linear algebra.
8. Prove various theorems/results involving any of the topics in (1) through (6) above using accurate mathematical language and notation to communicate arguments clearly.
9. Use technology such as graphing calculators and/or computer software to assist in solving problems involving any of the topics in (1) through (7) above.

Course Content

1. Solve linear systems using various methods in linear algebra, and analyze the systems
   a. Solutions of linear systems
   b. Elementary row operations and row echelon and row reduced echelon forms
   c. Gaussian elimination method
   d. Existence and uniqueness questions about a linear system
   e. Parametric descriptions of solution sets
   f. Vector equations
   g. Matrix equations
   h. Matrix transformations
   i. Linearly independent and dependent sets in R^n
   j. Linear transformation from R^n to R^m and its standard matrix
   k. One-to-one and onto linear transformations
2. Demonstrate an understanding of matrix operations, their properties, and various characterizations of invertible matrices including the Invertible Matrix Theorem
   a. Matrix operations and their properties
   b. Special types of matrices
   c. Transpose of a matrix
   d. Elementary matrices
   e. Inverse of a matrix and its properties
   f. Invertible Matrix Theorem
3. Evaluate the determinant and demonstrate an understanding of its properties
   a. Definition and properties of determinants
   b. Cofactor expansions
   c. Row operations and evaluation of determinants
4. Demonstrate an understanding of vector spaces and subspaces, identify those spaces and understand their characterizations
   a. Vectors in R^n space
   b. Abstract vector spaces and subspaces
   c. Special subspaces of a matrix: Null space, Column space, and Row space
   d. A subspace spanned by a set
   e. Linear transformation from a vector space to a vector space
   f. Kernel and range of a linear transformation
   g. Linearly independent and dependent sets in a general vector space
   h. Basis and dimension of a vector space
   i. Change-of-coordinates matrix
   j. Coordinate mapping

   k. Change of basis
   l. The rank of a matrix and the Rank Theorem
5. Demonstrate an understanding of eigenvectors, eigenvalues, and their usage in many fields
   a. Eigenvalue, eigenvector, and eigenspace of a matrix
   b. The characteristic equation
   c. Similar matrices and eigenvalues
   d. Diagonalization of a matrix
   e. Application to discrete dynamical systems
6. Demonstrate an understanding of an orthogonal projection of a vector onto a subspace, and solve related problems in linear algebra
   a. Inner product on R^n space
   b. Norm of a vector in R^n space
   c. Orthogonality of two vectors in R^n space
   d. Orthogonal complements
   e. Orthogonal sets and orthonormal sets
   f. Orthogonal basis and orthonormal basis
   g. Orthogonal projection of a vector onto a vector space
   h. Orthogonal decomposition
   i. Gram-Schmidt process
   j. Least-squares solution
   k. General inner product space
   l. Orthogonal diagonalization of a symmetric matrix
7. Write linear systems to model phenomena from various real-life problems, and discuss their solutions to demonstrate an understanding of applications of linear algebra
   a. Linear models from real-world applications
   b. Analyze and interpret the solution set in given contexts
8. Prove various theorems/results involving any of the topics in (1) through (6) above using accurate mathematical language and notation to communicate arguments clearly
9. Use technology such as graphing calculators and/or computer software to assist in solving problems involving any of the topics in (1) through (7) above

Lab Content

Not applicable.

Special Facilities and/or Equipment

Graphing calculator/Mathematica/MATLAB.

Method(s) of Evaluation

Methods of Evaluation may include but are not limited to the following:

Written homework and/or projects
Quizzes, tests
Proctored comprehensive final examination

Method(s) of Instruction

Methods of Instruction may include but are not limited to the following:

Lecture
Discussion
Cooperative learning exercises

Representative Text(s) and Other Materials

Lay, David C.. Linear Algebra and its Applications, 6th ed. (ISBN: 013588280X). 2021.

Types and/or Examples of Required Reading, Writing, and Outside of Class Assignments

1. Homework problems covering subject matter from text and related material ranging from 30-60 problems per week. Students will need to employ critical thinking in order to complete assignments.
2. Five hours per week of lecture covering subject matter from text and related material. Reading and study of the textbook, related materials and notes.
3. Student projects covering subject matter from textbook and related materials. Projects will require students to discuss mathematical problems, write solutions in accurate mathematical language and notation and interpret mathematical solutions. Projects may require the use of a computer algebra system such as Mathematica or MATLAB.
4. Worksheets: Problems and activities covering the subject matter. Such problems and activities will require students to think critically. Such worksheets may be completed both inside and/or outside of class.

Discipline(s)

Mathematics